FRACTAL METHODS APPLIED TO DESCRIBE CELLULAR AUTOMATON COMBAT MODELS

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This paper appeared in Vol 9, No 2, 2001.

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August, 2000

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Abstract

Analysis of warfare data provides compelling evidence that intensity of conflicts obeys a power law (fractal) dependence on frequency. There is also evidence for the existence of other power-law dependences and traits characteristic of high-dimensional chaotic systems, such as fat-tailed probability distributions and intermittency in warfare data. In this report, it is discussed how a cellular automaton model used to describe modern manoeuvre warfare produces casualty distributions which exhibit these properties. This points to a possible origin of the characteristics of the larger timescale data. More interesting, the techniques of fractal analysis offer a method by which to characterise these behaviours, and to quantify the difference between models based on complexity theory (such as cellular automata models), and more traditional combat models based on the physics of military equipment.

1. Introduction

Application of fractal methods to describe the statistical distribution of the intensities of wars has been discussed in this journal by Roberts and Turcotte[1]. Their analysis showed that fractal dimensions exist for such data, confirming the observation of Richardson that warfare statistics obey power laws[2,3].

Other types of power laws have also been identified in combat data, such as a non-integer power-law relationship between infantry force ratio (i.e. the number of attackers to defenders) and attack casualties[4,5]. These relationships have an interesting and natural geometric interpretation in terms of the fractal formalism[6].

New Zealand Defence Force's science organisation, the Defence Operational Technology Support Establishment, has been comparing the degree to which conventional combat models versus the methods of complexity theory are able to produce statistics that resemble these results[7].

In particular, investigations have centred on a cellular automaton model called ISAAC (Irreducible Semi-Autonomous Adaptive Combat)[8]. The last two decades have seen a growing interest in the use of cellular automaton models to describe a variety of problems, such as turbulence, forest fires[9,10,11,12], traffic jams[13,14,15], sand piles[16], and air traffic[17], particularly after the work on "self-organised criticality" by Bak *et al.*. Such models have also caught the attention of defence technology agencies[18,19,20,21,22].

The ISAAC model was developed as part of the US Marine Corp Combat Development Command's Project Albert[23] by the Center for Naval Analyses. Weapon systems are not explicitly described within the model, which relies on a high level of abstraction. Rather, the model is intended as a "distillation" of the key ingredients of

combat. Yet despite this simplicity, the model is capable of demonstrating a high degree of complexity.

The failure of more conventional combat models, despite their high level of physical detail, to describe the ability of units to react and organise themselves to fit a particular situation is a serious short-coming in representing reality. This leaves the analyst with a very precise picture of the outcome of one particular battle fought in a few particular ways with certain pieces of equipment modelled to some approximation of reality. Indeed, a growing number of military experts[24,25,26] have shown frustration at the fire-power centric models which completely fail to explore how a poorly equipped army might adapt to prevail against a superiorly equipped foe, as in Vietnam.

The difficultly cellular automaton, and more generally "multi-agent" models, face in gaining acceptance in the military operational research community is that it is hard to justify why they should be preferred over the more comfortable and predictable physics-based models. In short, there needs to be a better theoretical understanding of the properties of automaton models before they can gain wider acceptance. The work in this paper outlines how the methods of fractal analysis are proving a useful tool for characterising these differences, and furthermore, improving the understanding of the interaction between automata entities.

2. Automaton Combat Model

Here, not much detail will be given of the ISAAC model. The reader is referred to Ilachinski[8] for more detail. Additionally, the model itself may be downloaded from CNA's Website (www.cna.org/isaac), which contains information on the workings and philosophy of the model.

Each automaton's behaviour is governed by a set of parameters that determine its propensity to move towards friendly or enemy units, and towards its objective. A further set of parameters act as conditional modifiers to this process. A cluster parameter "turns off" the automata's propensity to move towards friends above some maximum cluster size; another parameter prevents automata moving towards their objective without a minimum number of friendly units accompanying it; and another determines the minimum local numerical advantage a group of automata require before engaging the enemy in combat.

A final set of parameters describes the basic capabilities of automata, such as weapons range, sensor range, movement rate, single-shot kill probability, defensive factor and maximum number of simultaneous targets that can be engaged. The automata also possess the ability to communicate the position of enemy/friendly automata to friendly automata within their communications range.

With these parameters, a wide set of behaviours can be induced. Even so, the version of the model discussed here is somewhat limited, and is not intended to describe every aspect of a military operation. Already, several new models are under construction that will improve functionality. However, for the purposes of this report, ISAAC does show interesting behaviour, particularly for fluid situations resembling modern "manoeuvre warfare".

Fig. 1 illustrates such a generalised concept of operation. Here, the Blue force must manoeuvre its way through a dispersed Red force to reach some objective (at the bottom of the frame). Various elements of the Red force react to the Blue force as it comes within sensor range, and Blue in turn reacts as Red surrounds it. Table 1 contains the values of the ISAAC parameters used to produce this run.

The personality rules of the simulation impose structure on the distribution as automata are attracted towards/repelled by enemy/friendly automata. This leads to groupings of automata into a complicated hierarchy of clusters that may "cooperate" in attacks on Blue automata[27]. The degree of clustering on the battlefield can be measured by obtaining a fractal dimension for the distribution, using a standard box-counting technique[28] (using just the Red forces). The box-counting fractal dimension, *D*, is defined here as:

$$D = \lim_{d \to 0} \frac{\log N}{\log\left(\frac{1}{d}\right)} \tag{1}$$

(where *d* is the width of the box, and *N* the number of boxes required to cover all the automata on the "battlefield"). D is a power-law exponent, since:

$$N(d) = d^{-D} \tag{2}$$

In order to accurately obtain a fractal dimension representative of the distributions that evolve from a particular set of parameters, a large ensemble of distributions must be used, and an average *D* found. This technique reveals a short scaling range for which *D* was estimated as 0.70 for the case discussed.

An argument can be made that the function describing the attrition rate should then depend on D^7. In a fixed time interval, a Blue automaton's "sensors" on average map out an area (box) proportional to the speed it is travelling. The density of Red automata — if they are encountered — is related to the number of boxes of this size that actually contain automata. But this has a non-integer power law dependence on the size of the box, and hence on the time interval (i.e. $l \propto t$ if *l* is the width of the box, and *t* the time interval). In such a case, the attrition rate function depends on:

$$\frac{\Delta n_B}{\Delta t} = f(k_{red}, \Delta t^{-D} n_{box}) \qquad (3)$$

where n_B is the number of remaining Blue automata, k_{red} is the probability of a Red automaton killing a Blue automaton within its detection and firing range with a single shot, n_{box} is the average number of Red automata contained in just those boxes that contain automata, and the size of the box is the area an automaton typically maps out in a period Δt. If there exists an ensemble of runs for which the distribution of forces that evolves produce a similar value for D, we hypothesis that for this ensemble the distribution of casualties depends only on k and Δt^{-D}, and has statistical moments which scale like:

$$\left\langle \left(\frac{\Delta n_B}{\Delta t} \right)^p \right\rangle \sim k^{q(D,p)} \, \Delta t^{r(D,p)} \qquad (4)$$

where q and r are non-integer powers related to D, the angled brackets denote an ensemble average, and the ensemble is conditional on Blue encountering Red in clusters of size n_{box}.

3. Properties of Model Data

Fig 2. shows the probability density for the number of Blue casualties suffered by the 500th time step of the model run shown in Fig. 1. The distribution is based on 2000 runs. The important point is that by this time step (and indeed, well before), the battle has run its course, so that further casualties are unlikely (i.e. Red and Blue are no longer interacting). Casualties do not necessarily reach 100 per cent (or some other arbitrary level), because the Blue force often breaks through to its goal, or otherwise avoids encountering concentrations of Red. Thus the distribution represents the

probability of ending a run in a particular state. Moreover, Blue only reaches a given level of casualties for a certain ensemble of runs, and these runs represent the occasions where Red was able to concentrate its firepower sufficiently to inflict this level. We may explore the nature of this ensemble by measuring attrition rate in terms of time taken to reach that level of casualties, excluding runs that do not.

Fig. 3 plots attrition rate as a function of k, calculated in this way, for the ensemble of runs with greater than 25 per cent Blue casualties. A further constraint on the ensemble was that this casualty level had to be reached during the manoeuvring stage of the simulation. Because the Blue automata stick close to the goal once they reach it, they are vulnerable to further attack if Red units drift down into this area. However, this is not the situation we are trying to model, so the arguments above do not apply. Hence where 25% casualties are reached as a result of this drift down behaviour, the runs were ignored.

The values for q may be found by fitting straight lines on the log-log plot of attrition rate versus k. Here the value for q was approximately 0.25. Note that above a certain value for k, the power law breaks down, with the slope tending to zero. This represents the case where the kill probability is sufficiently high that Blue is being killed almost immediately after encountering Red, so that increasing k has little further effect.

By contrast, a distribution of casualties from a case where each side is simply lined up to "shoot it out" is also shown in Fig. 2. Note that most conventional combat models are little more sophisticated than this approach[29,30,31,32,33]. A spread of outcomes is expected after a given period of time, but the level of casualties at that point represents a "snapshot" of casualty levels on the way to some arbitrary withdrawal

level. The fixed time interval used to produce the distribution shown in the plot was the mean time required to run the model to reach the same mean casualty level as for the fractal distribution case shown Fig. 1. Clearly, this distribution of casualties was quite different from the case where the forces had a fractal distribution, with the fractal case having a much longer "tail".

Fig. 3 also includes the attrition rate as a function of k for the "shoot it out" case. Note that for the ensemble used for this case, every run reached 25% casualties (though, in principle, some cases never reach this level, if, say, one side has sufficient luck that it eliminates its opponents before this happens). The value of q for this data was approximately 0.9.

In the previous section, it was supposed that the value of q depended on the fractal dimension of the distribution of forces. There appears to be evidence for this in other studies conducted with this model. For example, a similar operational scenario to that described above consisted of a Blue reconnaissance force represented by four Blue automata operating against a Red counter-reconnaissance force of 16 automata[34]. The Red force was modelled with and without communications. Clearly, communications has an effect on the degree to which Red is able to cluster to concentrate its firepower, and hence on the fractal dimension of the distribution of forces. The q values for the two cases were different, with attrition rate falling less rapidly as the value of k was lowered for the case where communications were on than when they were off.

Eq. (4) in the previous section implied that the attrition function also possesses fractal scaling structure. Fig. 4 shows the number of casualties per unit time step for the fractal case and for the shoot-it-out case. Here, in order to illustrate the temporal structure of the attrition function, it was necessary to increase both the number of

automata involved to 60 on each side, and the kill probabilities of each side (otherwise, the casualty data would be too sparse). For the fractal case, the casualties occur in a burst-like, non-systematic manner. By comparison, the shoot-it-out case produces a gradually declining attrition rate as the number of automata falls.

Fig. 5 shows how the second-order moments of the data scale as a function of t. The plot is actually the average of 40 plots produced from 40 runs of each case. The difference in the power laws produced reflects the difference in the temporal structure. For the fractal case, r (D, 2) was –0.24, while for the "shoot out" case the scaling did not produce the required straight line.

4. Conclusions

The main result presented here is that automaton combat models can generate fractal distributions of forces, which in turn affect the attrition rate of the automata. In particular, the non-integer power law that describes the attrition rate of certain ensembles seems to result from these fractal distributions, and is extremely significant for combat analysis.

It is tempting to see a relationship in the data suggesting that $q\ (D, 2) \approx -\ r\ (D, 2) \approx 1 - D$. However, the values obtained for q, r and D were some 10 to 20% out for this relationship to hold. While this might simply reflect that the estimates for the fractal dimensions were poor for this data owing to the size of the datasets, or other equally practical problems affecting their estimation, it seems more likely that if this relationship does indeed hold, it only does so for some idealised ensemble. On the whole, the ISAAC model produces the fractal behaviour described for only brief periods of the run. The fractal distribution might be interpreted as a case of self-

organised criticality, as has been claimed to exist for other automaton models. However, the rule set tends to cause the behaviour to "collapse" into non-complex situations, where, for example, the automata simply form a single clump at some end point.

In terms of combat modelling, the implications of these results are that a Red force with a relatively poor kill probability may do much better than suggested by a straight shoot-out approach for certain ensembles of runs. This is explained by the tendency of the automata to cluster together to concentrate firepower, rather than simply being the result of a series of "lucky shots", as would otherwise have to be the case.

The behaviour implies that for these models temporal and spatial correlations are critical and fat-tailed distributions are prevalent. Although degree of intermittency was not examined specifically in this report, there is also evidence that it plays an important role for this model[6,35].

This may go some way to explaining the occurrence of power laws in real combat data.

What is particularly useful about this behaviour is that the attrition function may be suitable to be described by the same sort of fractal cascade models that have been successfully used to describe turbulence and other kinds of geophysical signals[36,37]. It also raises the question of what the suitability of cascade models to describe this sort of automaton model says about the broader use of cascades models. Specifically, are cascade models simply a useful statistical "trick" to generate scaling data, or are they a valid physical model in their own right? Clearly, for the model examined here, no explicit "top down" cascade process occurs. Rather, it is a "bottom up" process. However, a casual observer with no knowledge of the model might speculate that a

cascade process was in operation, with clusters of automata splitting into smaller clusters, which themselves split into smaller clusters, etc, as the battle evolves. Perhaps, it is a matter of perspective.

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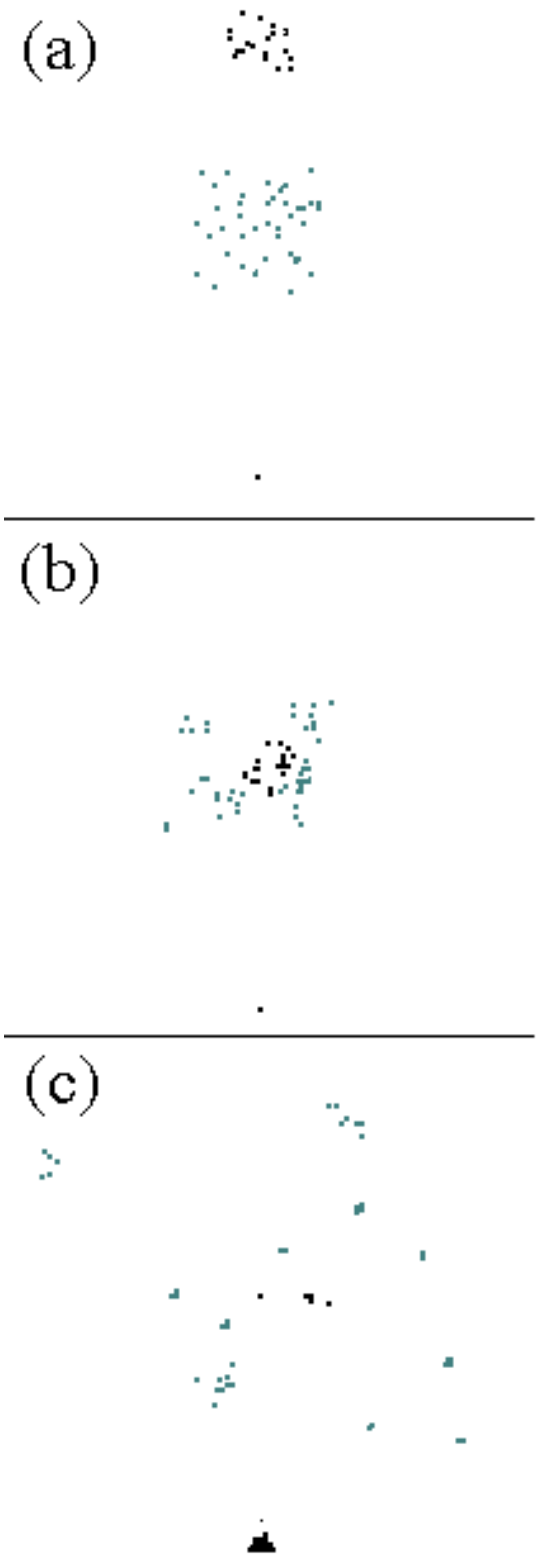

Figure 1: An evolutionary snapshot of an ISAAC run.

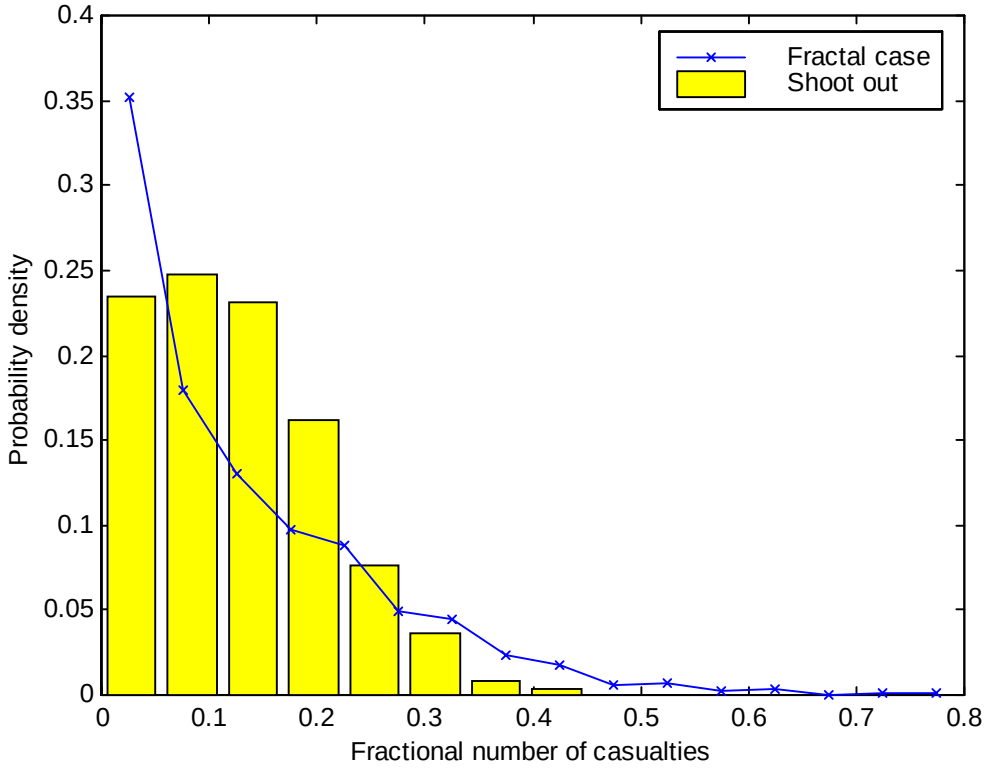

Figure 2: Casualty-level probability densities for the fractal and shoot-out cases. Each distribution has a mean of 0.15.

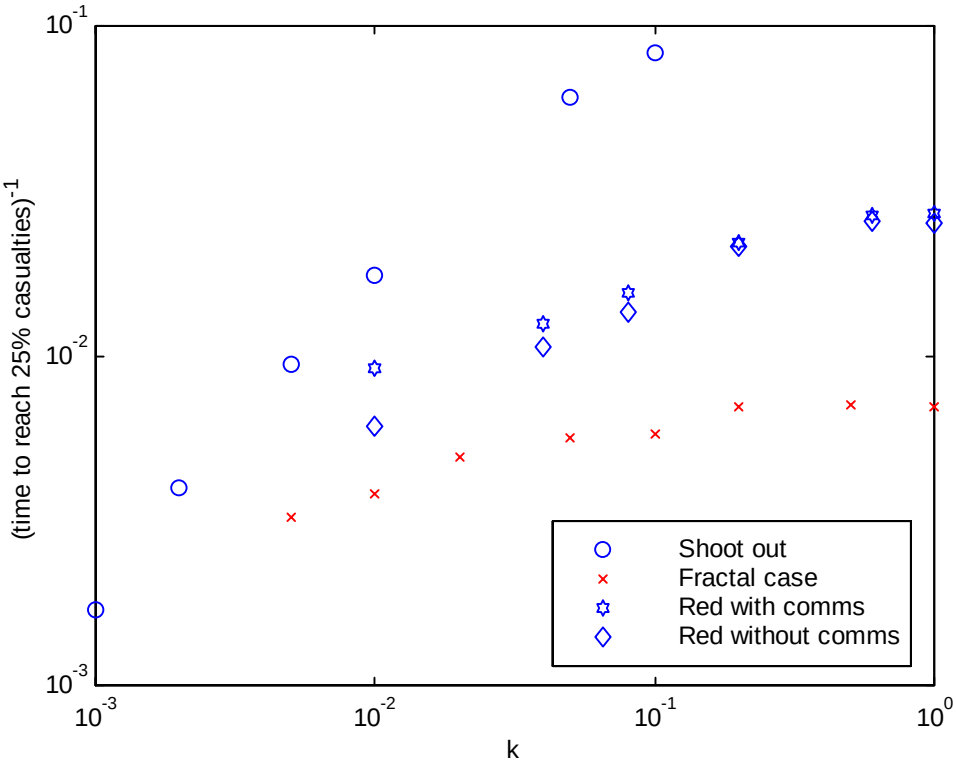

Figure 3: Attrition as a function of *k* for the ensemble of runs which reached 25% Blue casualties for various cases.

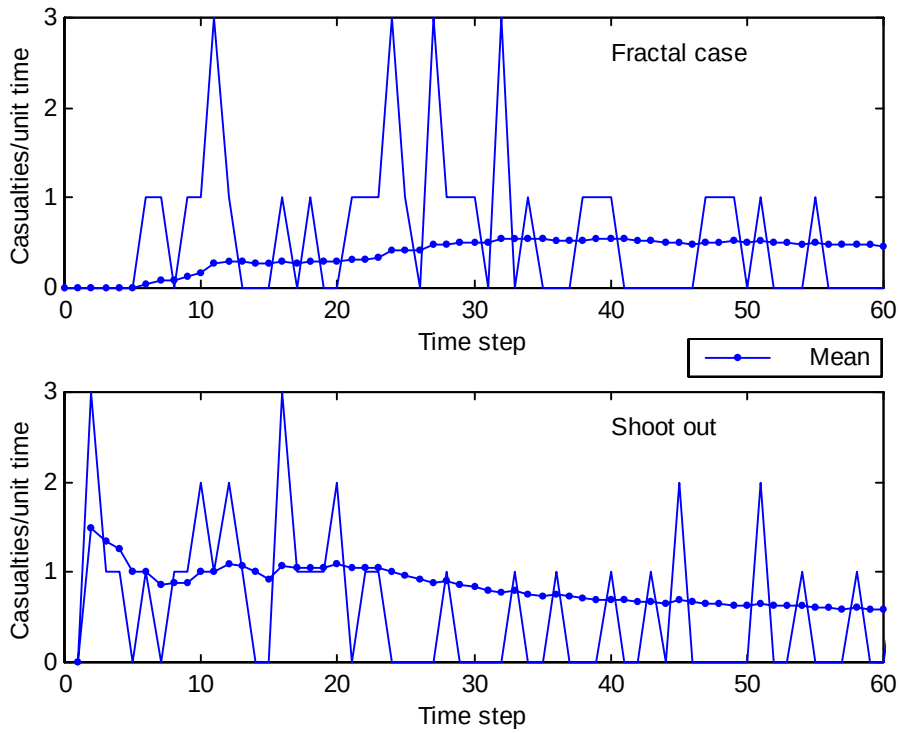

Figure 4: Time series of casualties and mean casualty rate for fractal and shoot-out cases.

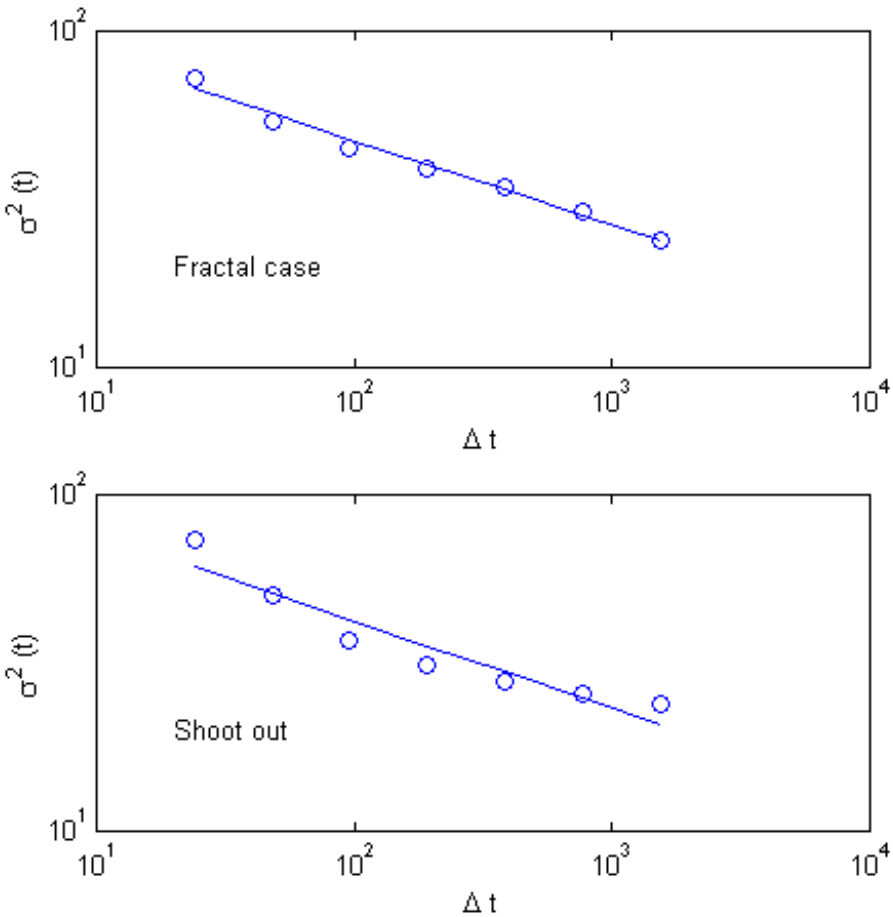

Figure 5: Scaling of the second-order moment of the time series data shown in Fig. 4.

	Personalities: weighting towards ... (threshold range = 5)			
	Alive/Injured Red	Alive/Injured Blue	Red flag	Blue flag
Red	10	40	0	0
Blue	40	10	10	0
	Meta personalities			
	Cluster	Advance	combat	
Red = Blue	3	1	3	
	Attributes			
	Fire range	Sensor range	Movement rate	*k* (baseline)
Red = Blue	3	5	1	0.05
	Initial distribution (battlefield size 120x120)			
	Centre position		Size	
Red	60,60		30,30	
Blue	60,105		15,15	
Red flag	60		1	

Table 1: ISAAC parameters for run shown in Fig. 1.